

## Overview of Calculating System Minimum Detectable Signal

### White Noise Contribution

There are a wide range of applications for which it is desirable or necessary to know the minimum power level at which a system receiver can detect an incoming signal. Because the minimum detectable signal (MDS) also pertains to the noise floor of system receiver, this is of particular import when the received power level may be close to the MDS. Knowledge of the MDS is also necessary when calculating the dynamic range or signal to noise ratio for a particular receiver configuration. The following treatment outlines a straightforward method to derive the MDS.

White noise is the blanket noise, containing a uniform power density per unit frequency interval, that is present in all electrical system. It can be shown that the noise power contained in an millimeter-wave receiver is given by Boltzman's Law, i.e.,

$$\text{NoisePower} = kTb \quad (1)$$

$$k = \text{Constant}$$

$$T = \text{Absolute Temperature (Kelvin)}$$

$$b = \text{Measurement Bandwidth}$$

In 1 Hz of measurement bandwidth, and at room temperature (293 Kelvin), the noise power contribution to the system receiver by white noise is:

$$kTb = -174 \text{ dBm}. \quad (2)$$

Rather than recalculating the noise power for other arbitrary measurement bandwidths using (1) above, a simplified method, utilizing (2), can be used.

MDS = Noise Floor = -174 dBm + Additional Noise Power in N Hz Bandwidth.

$$(3) \text{ Consider the logarithmic (dB) relation between a measurement bandwidth of 1 Hz and 1 MHz (} 10^6 \text{ Hz); } \text{dB}^1_{(1 \text{ MHz to } 1 \text{ Hz})} = 10 \log \left( \frac{10^6 \text{ Hz}}{1 \text{ Hz}} \right) = 60 \text{ dB}. \quad (4)$$

<sup>1</sup> This value can be arrived at quickly for any magnitudes of 1 Hz by taking the exponent of the measurement bandwidth and multiplying by ten (In the example above 6 → exponent times to = 60).

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Using (3), the additional noise power in a 1 MHz measurement bandwidth over that in its 1 Hz counterpart is:

$$\begin{aligned} kT\beta_{1 \text{ MHz Bandwidth}} &= kT\beta_{1 \text{ Hz Bandwidth}} + \text{Additional Noise Power in 1 MHz} = -174 \text{ dBm} + 60 \text{ dB} \\ &= -114 \text{ dBm in 1 MHz measurement bandwidth.} \end{aligned}$$

Similarly, in 1 GHz ( $10^9$  Hz) of measurement bandwidth:

$$\begin{aligned} kT\beta_{1 \text{ GHz Bandwidth}} &= kT\beta_{1 \text{ Hz Bandwidth}} + \text{Additional Noise Power in 1 GHz} = -174 \text{ dBm} + 90 \text{ dB} \\ &= -84 \text{ dBm in 1 GHz measurement bandwidth.} \end{aligned}$$

### Receiver Noise Contribution

The contribution to the system noise power by the receiver is found by determining the system noise figure (NF). For receivers with amplifiers, the receiver noise figure is given as;

$$NF_{\text{Receiver}} = NF_{1 \text{ Component}} + \frac{NF_{2^{\text{ND}} \text{ Component}}}{\text{Gain}_{1^{\text{st}} \text{ Component}}} + \frac{NF_{3^{\text{rd}} \text{ Component}}}{\text{Gain}_{1^{\text{st}} \text{ Component}} * \text{Gain}_{2^{\text{nd}} \text{ Component}}} + \dots \quad (5)$$

Typically, for amplifiers with gains of greater than 13 dB, noise figure terms following the amplifier can be discounted<sup>2</sup>.

For most millimeter-wave components, the insertion loss or conversion loss is specified but the noise figure is not. At room temperature (293 K), the component noise figure can be taken as equal to the insertion loss or conversion loss value. For other ambient temperatures, the noise temperature of the component can be determined via;

$$\begin{aligned} T &= (L-1)T_{\text{AMB}} + \\ T &= \text{Component Noise Temperature} \\ T_{\text{AMB}} &= \text{Ambient Temperature in which component operates} \\ L &= \text{Fractional Loss of Component} \end{aligned}$$

This can be converted into a NF value using;

$$NF_{\text{Component}} = 10 \log \left[ \left( \frac{T_{\text{Component}}}{295 \text{ K}} \right) + 1 \right]. \quad (6)$$

A [table](#) of noise temperature to noise figure conversions can be used.

The receiver noise figure can be found by adding the component contributions using (5) above.

<sup>2</sup> Consider an input amplifier with 13 dB of gain.  $13 \text{ dB} \equiv 20$  that, when substituted for  $\text{Gain}_{1^{\text{st}} \text{ Component}}$  in (4) yields a second component noise figure contribution of  $\frac{NF_{2^{\text{nd}} \text{ Component}}}{20}$ .

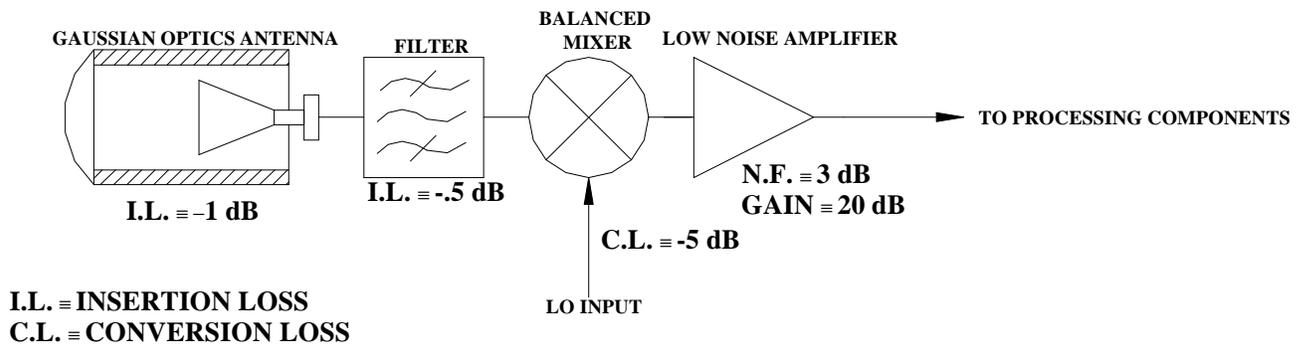
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**Receiver Minimum Detectable Signal (Noise Floor)**

The MDS or noise floor for any receiver system can then be found by adding the noise power contributions obtained from (3) and (5) above.

**Example:**

The system diagrammed below resides at room temperature. The measurement bandwidth is



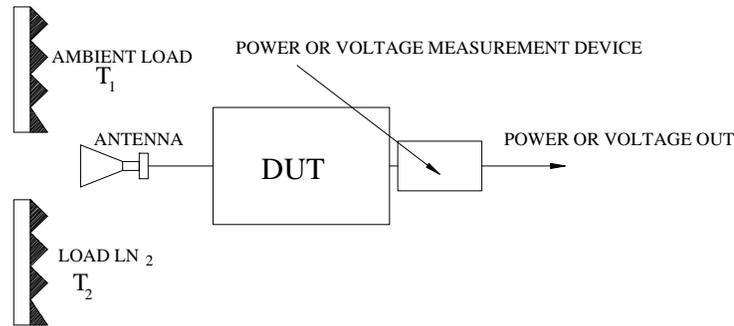
100 kHz

The relative noise power at a measurement bandwidth of 100 kHz to that at 1 Hz is 50 dB ( $100 \text{ kHz} = 10^5$ ) yielding a white noise power contribution of  $-174 \text{ dBm} + 50 \text{ dB} = -124 \text{ dBm}$ . Using the insertion loss or conversion loss to noise figure equivalence approximation at room temperature and adding for the components in the diagram yields a receiver noise figure of  $1.0 \text{ dB} + 0.5 \text{ dB} + 5.0 \text{ dB} + 3 \text{ dB} = 9.5 \text{ dB}$ <sup>3</sup>. Adding these two values yields an MDS or noise floor value of  $-114.5 \text{ dBm}$ .

<sup>3</sup> The approximation on post amplifier noise power contributions has been ignored due to the high gain of the amp.

## A Brief Description of Y-Factor Radiometric Measurements

A generic test setup is shown below.



**Figure 1: Measurement Test Setup**

The receiver/radiometer<sup>4</sup> is allowed to reach a thermal steady state of  $T_{AMB1}$ . A load consisting of absorbing material (carbon based) at  $T_1$  (known ambient laboratory temperature) serves as the hot calibration load while the same absorbing material, bathed in liquid Nitrogen ( $LN_2$ ;  $T_2 = 77$  K), serves as the cold load. The absorbing material, residing at  $T_{HOT}$  ( $T_{COLD}$ ) and then  $T_{COLD}$  ( $T_{HOT}$ ) ( $T_1$  and  $T_2$  in Figure 1 above), is placed in front of the receiver/radiometer such that it fills the entire field of view of the antenna. The voltage<sup>5</sup> or power output of the measurement device is then recorded for each temperature yielding:

$$P_{hot} = \text{Power measurement at ambient load temperature} = V_{HOT}$$

$$P_{cold} = \text{Power measurement at } LN_2 \text{ load temperature.} = V_{COLD}$$

The delta Y-Factor value is obtained from the slope,  $dY$ , of the graph of the temperatures verses their respective voltages depicted in Figure 2 below.

The slope of the graph can be written in terms of the hot and cold Y-Factor temperatures and the temperature of the device under test via the following equation:

$$dY = \frac{P_{HOT}}{P_{COLD}} = \frac{T_{HOT} + T_{DUT}}{T_{COLD} + T_{DUT}}; T_{HOT} = T_1, T_{COLD} = T_2.$$

Solving for  $T_{DUT}$  yields:

$$T_{DUT} = \frac{T_{HOT} - dY \cdot T_{COLD}}{dY - 1}$$

<sup>4</sup>  $T_{MIXER}$  will be used to denote the noise temperature of receiver, radiometer or device under test.

<sup>5</sup> A detector operating in the square law region yielding a voltage that is directly proportional to the input power.

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The noise figure can then be calculated from the DUT noise temperature via:

$$NF_{DUT(dB)} = 10 \log \left( \frac{T_{DUT}}{295} + 1 \right)$$

It is important to note that the above calculation does not take into account the noise added by the antenna or by any other device in the measurement chain.

Y-Factor Measurement

